

# Hybrid Neural Networks for Frequency Estimation of Unevenly Sampled Data

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## Abstract

*In this paper we present a hybrid system composed by a neural network based estimator system and genetic algorithms. It uses an unsupervised Hebbian nonlinear neural algorithm to extract the principal components which, in turn, are used by the MUSIC frequency estimator algorithm to extract the frequencies. We generalize this method to avoid an interpolation preprocessing step and to improve the performance by using a new stop criterion to avoid overfitting. Furthermore genetic algorithms are used to optimize the neural net weight initialization.*

*The experimental results are obtained comparing our methodology with the others known in literature on a Cepheid star light curve.*

## 1. Introduction

Periodicity analysis of unevenly collected data is a relevant issue in several scientific fields. Classical spectral analysis methods are unsatisfactory to solve the problem. In this paper we present a neural network based estimator system which performs well the frequency extraction in unevenly sampled signals. It uses an unsupervised Hebbian nonlinear neural algorithm to extract the principal components of the signal auto-correlation matrix, which, in turn, are used by the MUSIC frequency estimator algorithm to extract the frequencies [6], [18]. We generalize this method to avoid an interpolation preprocessing step, which generally adds high noise to the signal, and improve the system performance by using a new stop criterion to avoid overfitting problems. Furthermore, genetic algorithms are used to optimize the neural net weight initialization, so improving its convergence. The experimental results are obtained comparing our methodology with the others known in

literature [7],[13],[9],[17],[3],[14].

## 2. Evenly and unevenly sampled data

In what follows, we assume  $x$  to be a physical variable measured at discrete times  $t_i$ .  $x(t_i)$  can be written as the sum of the signal  $x_s$  and random errors  $R$ :  $x_i = x(t_i) = x_s(t_i) + R(t_i)$ . The problem we are dealing with is how to estimate fundamental frequencies which may be present in the signal  $x_s(t_i)$  [2], [7].

If  $X$  is measured at uniform time steps (even sampling) [4],[17] there are a lot of tools to effectively solve the problem which are based on Fourier analysis [7],[13]. These methods, however, are usually unreliable for unevenly sampled data. For instance, the typical approach of resampling the data into an evenly sampled sequence, through interpolation, introduces a strong amplification of the noise which affects the effectiveness of all Fourier based techniques which are strongly dependent on the noise level [5].

To solve the problem of unevenly sampled data, we consider two classes of spectral estimators:

Spectral estimators based on Fourier Transform (Least Squares methods);

Spectral estimators based on the eigenvalues and eigenvectors of the covariance matrix (Maximum Likelihood methods).

Classic Periodogram [7],[13], Lomb's Periodogram [9], Scargle's Periodogram [17], DCDFFT [3] are the methods of the first class that we use, while MUSIC [7],[13] and ESPRIT [15],[14] belong to the second class.

The methods based on the covariance matrix are more recent and have great potentiality. Starting by this consideration, we develop a method based on the MUSIC

estimator. It is compared with classical methods to evidence the performance.

### 3. The Neural Estimator

In the last years several papers dealt with learning in PCA neural nets [11],[12],[16],[6],[1], [18] finding advantages, problems and difficulties of such neural networks. In what follows, we shall use a robust hierarchical learning algorithm because it has been experimentally shown that it is the best performing in our problem [18].

Our neural estimator (ne) can be summarized as follows:

- 1 Preprocessing: calculate and subtract the average pattern to obtain zero mean process with unity variance.

Interpolate input data if it is the case.

- 2 Train the neural network.

- 3 Calculate the frequencies estimation by using the frequency estimator MUSIC.

MUSIC takes as input the weight matrix columns of the neural network after the learning. The estimated signal frequencies are obtained as the peak locations of the function of the following equation [7]:

$$P_{MUSIC} = \frac{1}{M - \sum_{i=1}^M |\mathbf{e}_f^H \mathbf{w}(i)|^2}$$

where  $\mathbf{w}(i)$  is the  $i$ -th neural network weight vector after learning, and  $\mathbf{e}_f^H$  is the vector

$$\mathbf{e}_f^H = [1, e^{j2\pi f t_0}, \dots, e^{j2\pi f t_{(L-1)}}]^H$$

where  $\{t_0, t_1, \dots, t_{(L-1)}\}$  are the first  $L$  components of the temporal coordinates of the uneven signal [18].

When  $f$  is the frequency of the  $i$ -th sinusoidal component,  $f = f_i$ , we have  $\mathbf{e} = \mathbf{e}_i$  and  $P_{MUSIC} \rightarrow \infty$ . In practice we have a peak near and in correspondence of the component frequency. Estimates are related to the highest peaks

Furthermore, to optimize the performance of the PCA neural networks, we stop the learning process when

$$\sum_{i=1}^p |\mathbf{e}_f^H \mathbf{w}(i)|^2 < M \quad \forall f,$$

so avoiding overfitting problems. In fact, leaving the stop condition used in the ne causes to the ne to find

periodicities not present in the signal, while the new condition preserves it from this problem.

Since sometimes the weight initialization can lead to local maxima of the objective function, here we propose to use genetic algorithms for this aim.

We use a real encoding of the net parameters [8] in a string (chromosome), and then we run the genetic algorithms with a fitness function, which is the system objective function.

### 4. Experimental Results

Many experiments on synthetic and real signals were made [18], and in this paper we presents the results obtained with one specific real signal, which highlights the main features of our problem.

The signal is related to the Cepheid T Mon [10]. The sequence was obtained with the photometric technique BVRI and the sampling made from April 1977 to December 1979. The light curve is composed by 24 samples, and a period of  $27.02^d$ , as shown in figure 1. In this case, the parameters of the ne are:  $N = 10$ ,  $p = 2$ ,  $\alpha = 5$ ,  $\mu = 0.001$ . The estimated frequency interval is  $[0(1/JD), 0.5(1/JD)]$ . The estimated frequency is  $0.020(1/JD)$  (see figure 2). By using the genetic algorithms (gne) we improve the system performance reaching the true frequency of  $0.034(1/JD)$  (see figure 3). Lomb's Periodogram finds the correct frequency but it generates several spurious peaks (see figure 4). For what concerns DCDF, the frequency interval must be changed because it is too much sensitive to very low frequencies (the new interval becomes  $[0.01(1/JD), 0.5(1/JD)]$ ) (see figure 5). Finally ESPRIT does not work at all finding a frequency of  $0.23(1/JD)$  (see figure 6).

In conclusion, only the Lomb's Periodogram and our gne are in agreement with the right periodicity, but the former showing several spurious peaks.

### 5. Concluding Remarks

In this paper we have illustrated an improved technique based on PCA neural networks and MUSIC to estimate the frequency of unevenly sampled data. It has been shown that it obtains good results on real data (here we used the Cepheid T Mon light curve) compared with other well known methods. A further improvement can be obtain by using filters to extract and identify one frequency at each time when dealing with multi-frequency

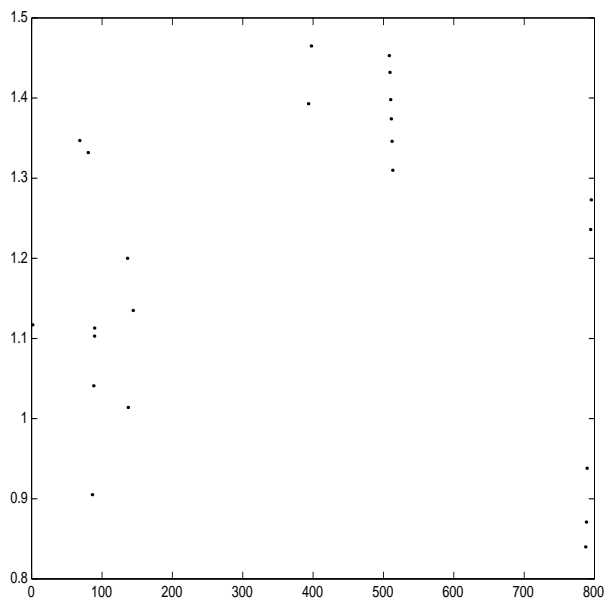


Figure 1: Light curve of T Mon

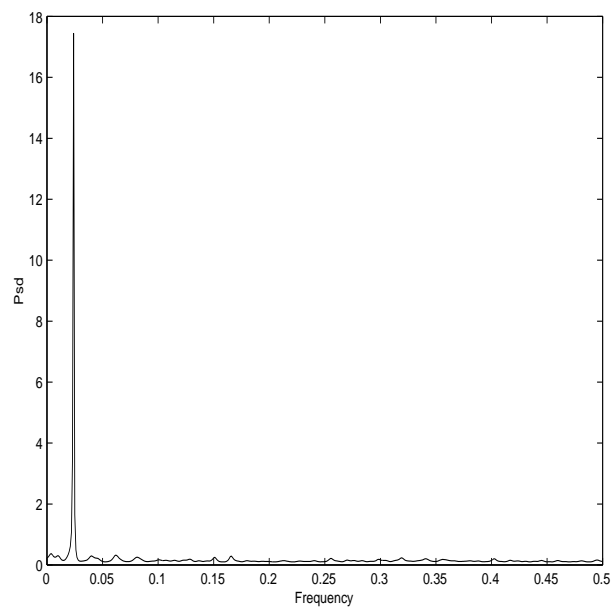


Figure 3: gne estimate of the fundamental frequency of T Mon

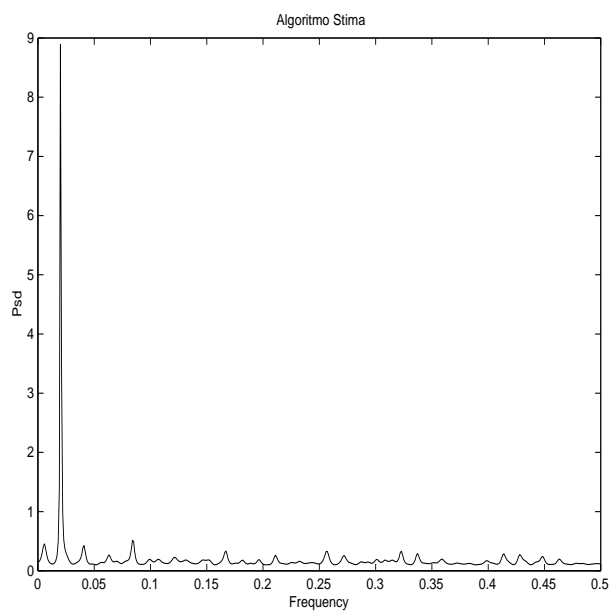


Figure 2: ne estimate of the fundamental frequency of T Mon

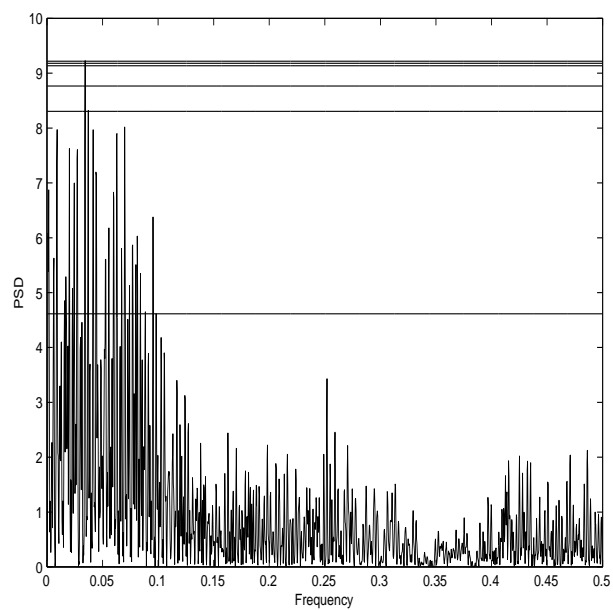


Figure 4: Lomb's Periodogram estimate of the fundamental frequency of T Mon

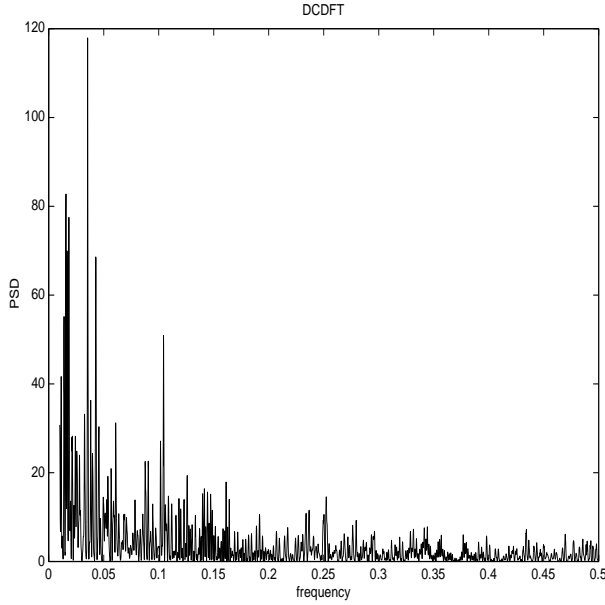


Figure 5: DCDFIT estimate of the fundamental frequency of T Mon

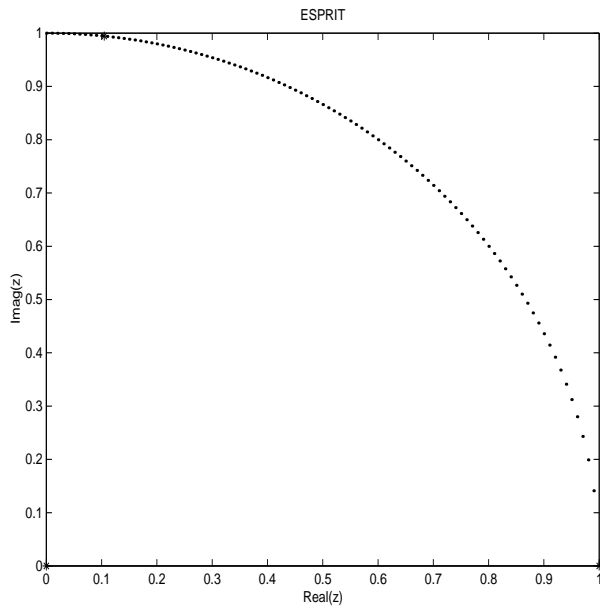


Figure 6: ESPRIT estimate of the fundamental frequency of T Mon

signals. In [19], we used our system to detect the Milankovic' frequencies from a stratigraphic record. In that case the best performance was found when we extracted one frequency at each time and eliminating it with a FIR filtering.

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